Towards Stochastic Delay Bound Analysis for Network-on-Chip

Zhonghai Lu*, Yuan Yao* and Yuming Jiang†

*KTH Royal Institute of Technology, Sweden
†Norwegian University of Science and Technology

The 8th IEEE/ACM International Symposium on Networks-on-Chip
September 17-19, 2014
Agenda

- Problem introduction
- Analysis
  - Router service modeling
  - Analysis flow
- Experiments
  - Tandem network
  - Video playback application
- Summary
The on-chip network

- Typically synchronous network/region
- Pure hardware router
- Virtual channels
- Clear-box router

Here flow is a unicast packet stream along a specific path.
The long tail of network delay

- Packet delay histogram of F0, marking also average delay

\[ \epsilon = \lambda / \mu \]

(a) \( \epsilon_1 = 0.2 \) \( (\lambda_1 = 0.1, \mu_1 = 0.5) \)

(b) \( \epsilon_0 = 0.5 \) \( (\lambda_0 = 0.1, \mu_0 = 0.2) \)

Rate of tag flow: 0.35 \( \Rightarrow \) 0.5
Rate of interference flow: 0.2

Rate of tag flow: 0.5
Rate of interference flow: 0.2 \( \Rightarrow \) 0.35
Key questions

• How long is the long tail?

• What is the exceeding likelihood of a particular (large) delay? Can it be bounded with a function?

• Simulation is time consuming and might be hard to answer the questions, especially for large networks.
• Simulation may be used as a complement technique for validation.
This paper is about

- Analysis of \textit{stochastic} delay which
  - is based on \textit{stochastic} network calculus
  - gives \textit{stochastic} delay bound

\textit{Yuming Jiang and Yong Liu. Stochastic Network Calculus. Springer 2008}

- This work is in contrast to worst-case delay analysis which
  - is based on \textit{deterministic} network calculus
  - gives \textit{deterministic} delay bound

Like a typical theoretical approach, we need

- Traffic model
- Server model
- Mathematical tool
- Metric of interest

Deterministic network calculus

• Traffic flow conforms to Arrival Curve, which defines the upper bound of cumulative traffic arrival process.
• Server conforms to Service Curve, which defines the lower bound of cumulative service process.

A linear arrival curve has the form

\[ \alpha(t) = \sigma + \rho(t) \]

where \( \sigma \) bounds burstiness, \( \rho \) average rate
General basic results

Assume a flow constrained by arrival curve $\alpha$ traverses a system offering a service curve $\beta$

- The delay for all $t$ is bounded by
  \[ D(t) \leq h(\alpha, \beta) \]

  $h(\alpha, \beta)$: the max. horizontal distance.

- The backlog for all $t$ satisfies
  \[ B(t) \leq v(\alpha, \beta) \]

  $v(\alpha, \beta)$: the max. vertical distance.
Traffic model

- Let \( A(t) \) be traffic arrival process. The cumulative amount of traffic from time \( s \) to \( t \) is

\[
A(s, t) = A(t) - A(s)
\]

- Stochastic arrival curve

\[
P\{\sup_{0 \leq s \leq t} \{A(s, t) - \alpha(t - s)\} > x\} \leq f(x)
\]

The probability of the maximum difference between the cumulative amount of traffic \( A(s, t) \) and a given function \( \alpha \) during any time interval \( t - s \) greater than \( x \) is not larger than \( f(x) \). We say \( A \sim < f, \alpha > \).
Compound Poison distribution

- Consider a Poisson process with density $\lambda$, and the packet lengths are independent and identically distributed, following a negative exponential distribution with mean $1/\mu$, the Poisson process has a stochastic arrival curve

- curve $\alpha(t) = \frac{\lambda}{\mu - \theta} t$ with

- bounding function $f(x) = e^{-\theta x}, \forall \theta > 0$
Service model

• Consider a system S with input arrival process A(t) and output departure process A*(t).

• **Stochastic Service Curve:** A server S provides a (weak) stochastic service curve, \( \beta \), with bounding function \( g \), denoted by \( S \sim < g, \beta > \) if \( \forall t \geq 0 \) and \( \forall x \geq 0 \),

\[
P\{A \boxtimes \beta(t) - A^*(t) > x\} \leq g(x)
\]

where \( \boxtimes \) is **min-plus convolution**, defined as

\[
(f \boxtimes g)(t) = \min_{0 \leq s \leq t} [f(s) + g(t - s)]
\]

\[
(f \boxtimes g)(t) = \sum_{s=0}^{t} f(s) \cdot g(t - s)
\]

- Plus => Min
- Multiplication => Plus
Stochastic delay bound

- Consider that node $S$ serves an input flow $A$. Suppose $A$ has a stochastic arrival curve $A \sim< f, \alpha >$, and $S$ a stochastic service curve $S \sim< g, \beta >$. Then $\forall t \geq 0$ and $x \geq 0$, the delay $D(t)$ of flow $A$ is probabilistically bounded by

$$P\{D(t) > h(\alpha + x, \beta)\} \leq f \otimes g(x)$$

where $h(\alpha+x, \beta)$ represents the maximum horizontal distance between the two curves, $\alpha+x$ and $\beta$. 
Analytical router modeling

- A virtual channel router
- Deterministic routing
- Static VC allocation
- FIFO queues
Router service modeling

- Each router has two service points to serve queued packets
  - De-multiplexing
    - At the crossbar input
    - Multiple servers
  - Multiplexing
    - At the crossbar output
    - Channel arbitration
The two-node basic case

- Per-flow router service modeling
- Example with 2 nodes 3 flows

Router – flow graph

NC analysis model
End-to-end stochastic delay analysis

- Per-flow analysis, end-to-end
  - By end-to-end, avoid to compute the departure curve at each router output
- The key step is to
  1. Per-router equivalent stochastic service curve (ESSC)
  2. End-to-end (e2e) stochastic service curve
- Stochastic arrival model + e2e stochastic service curve => per-flow stochastic delay bound
General analysis flow

1. Derive per-router ESSC for all routers visited by $Fi$.
2. Deduct per-path (end-to-end) ESSC for $Fi$.
3. Compute end-to-end stochastic delay bounding function.
Closed-form result

- Suppose that
  - Independent flows $F_i$ follows the compound Poisson process with intensity $\lambda_i$ and packet length mean $1/\mu_i$. Thus $F_i \sim < e^{-\theta_i x}, \frac{\lambda_i}{\mu_i - \theta_i} t >$, where $\theta_i > 0$.
  - Weighted round robin arbitration. $T_{w0}$ is the maximum time needed to serve the other flow per service round.
  - Per-hop propagation delay $T_{hop}$
  - Channel capacity $C$

- The stochastic packet delay bound for $F_0$ is

$$ P\{D(t) > x\} \leq (1 + \theta x') e^{-\theta x'} $$

where $\theta = \theta_0 \wedge \theta_1$ with $\theta_i = \mu_i - \lambda_i$ ($i = 0, 1$).

$$ x' = (x - 2(T_{w0} + T_{hop})) \cdot (w_0 C - \rho_1) $$
Generalization of the result

Consider flow $F_0$ passing a series of $N$ routers with the same interference pattern (in total $N+1$ flows), $F_0$’s stochastic delay bound is

$$P\{D(t) > x\} \leq f_0 \otimes f_{sys}(x_{sys})$$

where $\theta = \theta_0 \land \theta_1 \land \theta_2 \cdots \land \theta_{N-1}$ with $\theta_i = \mu_i - \lambda_i$ ($i \in \mathbb{N}_0$).

$$f_{sys} = f_1 \otimes f_2 \cdots \otimes f_{N-1}$$

$$x_{sys} = (x - N(T_{w_0} + T_{hop})) \cdot C_{sys}$$

$$C_{sys} = (w_0C - \rho_1) \land (w_0C - \rho_2) \cdots \land (w_0C - \rho_{N-1})$$
Evaluation

• Purpose
  - Accuracy
  - Scalability
  - Applicability

• Methodology
  - Compare F0’s simulated delay statistics with its theoretical delay bounds.
  - Explore the theoretical bound.
    • If/What can we gain if relaxing a deterministic delay bound to a probabilistic delay bound?
    • How much?
Experimental setup

- The tandem network

- Independent flows with Poison distribution
- Video playback experiment
Experiment 1: 2 nodes, 3 flows

Both phenomena indicate that, while the bounding function computes the CCDF under the consideration that interference always exists, higher likelihood of flow interference due to higher network utilization (indicated by larger $\varepsilon$) results in better tightness.

$\Delta_{max}$ from 43% to 37%
$\Delta_{avg}$ from 13.2% to 10.6%

$\Delta_{max}$ from 37% to 24%
$\Delta_{avg}$ from 10.6% to 2.1%
The analysis gives tighter results when the network utilization (indicated by $\varepsilon$) is higher.

When $F_0$ experiences more contention from more flows, the discrepancy $\Delta_{\text{max}}$ drops quickly. When $N = 14$ and $\varepsilon = 0.2$, $\Delta_{\text{max}}(14) = 0.046$ (4.6%).
Experiment 3: Explore theoretical results

- If we require that $P\{D(t) > 150\} \rightarrow 0$, $\varepsilon = 0.2$; If relaxing this requirement to $P\{D(t) > 150\} \leq 0.02$ (2%), network utilization $\varepsilon$ is improved from 0.2 to 0.33, i.e., 65% in percentage.
- Further relaxing this requirement improves $\varepsilon$ further but with lower acceleration. The maximum $\varepsilon$ is saturated till 0.38 when $P\{D(t) > 150\} \leq 0.1$ (10%).
Video playback application

- $F_i \ (i = 0, 1, 2, \ldots, 14)$ consists of video streams read from Tektronix MPEG2 source files and sent over the 14-node network to a video decoder at the destination.
- The video streams are encoded at 30 fps with different bit rates in Mbps. One packet contains constant bytes of data.
• The stream reading is controlled by a parameterized Poisson process. As a result, when the flows enter the network, they follow their respective Poisson distribution.
• The packet delay threshold is set to 150 cycles. A received packet is stored for playback if its delay is less than this threshold, and discarded otherwise.
• We record the number of played packets, which is converted to playback rate in fps.
At higher bit rates, some packets may experience a delay larger than 150 cycles, resulting in reduced playback rate.

Nevertheless, the network, which is designed for transferring a bit rate of 20 Mbps without delay violation ($\varepsilon = 0.2$), has negligible loss in playback rate at $\varepsilon = 0.33$, and can even transfer a bit rate of 44 Mbps ($\varepsilon = 0.375$) with only 4.67% reduction in video quality (from 30 to 28.6 fps).
Summary

- A study on stochastic delay bound
  - gives good estimates
  - shows promising insights on long tail, delay exceeding probability.

- Future work
  - Credit-based flow control, which is so far an open challenge in stochastic network calculus theory
  - Possibility to tighten the bounding function for many different models such as left-over service, multi-server model etc.